Q1

1

State whether the following mappings are one-to-one, many-to-one, one-to-many or many-to-many.

(i) $f: x \mapsto 2 - x^3$
(ii) $f: x \mapsto \sin x$
(iii) $f: x \mapsto \frac{1}{x^2}$
(iv) $f: x \mapsto \ln x$

[4]

i) ONE TO ONE

CUBIC

ii) MANY TO ONE

$\sin 30 = \frac{1}{2}$

$\sin 150 = \frac{1}{2}$

iii) MANY TO ONE

$\frac{1}{2^2} = \frac{1}{4}$ $\frac{1}{(-2)^2} = \frac{1}{4}$

iv) ONE TO ONE

NATURAL LOG

Q2a

2a

It is given

$f(x) = \frac{2}{x}$ RECIPROCAL

(a) Write down the domain of the function $f(x)$.

[1]

(b) Sketch the graph of $y = f(x)$, stating the coordinates of any intersections with the coordinate axes and the equations of any asymptotes.

[3]

(c) Write down the range of $f(x)$.

[1]

a) DOMAIN $x \in \mathbb{R}$ $x \neq 0$

Q2b

2b

It is given

$$f(x) = \frac{2}{x}$$

(a) Write down the domain of the function $f(x)$.

[1]

(b) Sketch the graph of $y = f(x)$, stating the coordinates of any intersections with the coordinate axes and the equations of any asymptotes.

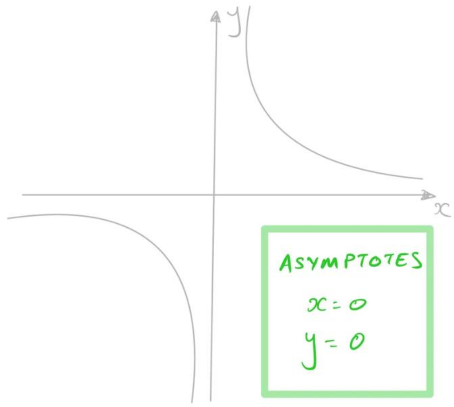
NONE

[3]

(c) Write down the range of $f(x)$.

[1]

b) RECIPROCAL GRAPH



Q2c

2c

It is given

$$f(x) = \frac{2}{x}$$

(a) Write down the domain of the function $f(x)$.

[1]

(b) Sketch the graph of $y = f(x)$, stating the coordinates of any intersections with the coordinate axes and the equations of any asymptotes.

[3]

(c) Write down the range of $f(x)$.

[1]

c) $\boxed{\text{RANGE } f(x) \in \mathbb{R} \quad f(x) \neq 0}$

Q3a

3a

The function $f(x)$ is defined as

$$f(x) = x(x+3)^2 + 1 \qquad x \geq 0$$

(a) Work out the range of $f(x)$. [1]

(b) If the domain of $f(x)$ is changed to $x \leq 0$, what is the range of $f(x)$? [2]

a) $\boxed{\text{RANGE } f(x) \geq 1}$

RANGE = OUTPUT
POSITIVE CUBIC
$x \geq 0$

Q3b

3b

The function $f(x)$ is defined as

$$f(x) = x(x+3)^2 + 1 \qquad x \geq 0$$

(a) Work out the range of $f(x)$. [1]

(b) If the domain of $f(x)$ is changed to $x \leq 0$, what is the range of $f(x)$? [2]

b) $\boxed{f(x) \leq 1}$

POSITIVE CUBIC
$x \leq 0$
GIVES NEGATIVE OUTPUTS

Q4a

4a

The functions $f(x)$ and $g(x)$ are defined as follows

$f(x) = 3x^2 + 2$ $\quad x \in \mathbb{R}$
$g(x) = 1 - 3x$ $\quad x \in \mathbb{R}$

(a) Write down the range of $f(x)$. [1]

(b) Find (i) $fg(x)$
(ii) $gf(x)$ [4]

(c) Solve the equation $f(x) = g(x) + 1$ [2]

a)

RANGE = OUTPUT

QUADRATIC
$3x^2$ ALWAYS POSITIVE

$\boxed{f(x) \geqslant 2}$

Q4b

4b

The functions $f(x)$ and $g(x)$ are defined as follows

$f(x) = 3x^2 + 2$ $\quad x \in \mathbb{R}$
$g(x) = 1 - 3x$ $\quad x \in \mathbb{R}$

(a) Write down the range of $f(x)$. [1]

(b) Find (i) $fg(x)$
(ii) $gf(x)$ [4]

(c) Solve the equation $f(x) = g(x) + 1$ [2]

COMPOSITE FUNCTIONS

b)

i) $fg(x)$

$3(1-3x)^2 + 2$

$3(9x^2 - 6x + 1) + 2$

$\boxed{fg(x) = 27x^2 - 18x + 5}$

ii) $gf(x)$

$1 - 3(3x^2 + 2)$

$1 - 9x^2 - 6$

$\boxed{gf(x) = -9x^2 - 5}$

Q4c

4c

The functions $f(x)$ and $g(x)$ are defined as follows

$f(x) = 3x^2 + 2 \qquad x \in \mathbb{R}$
$g(x) = 1 - 3x \qquad x \in \mathbb{R}$

(a) Write down the range of $f(x)$. [1]

(b) Find (i) $fg(x)$
(ii) $gf(x)$ [4]

(c) Solve the equation $f(x) = g(x) + 1$ [2]

c) $3x^2 + 2 = 1 - 3x + 1$

EQUATE AND SOLVE

$3x^2 + 3x + \cancel{2} = \cancel{2}$

$3x^2 + 3x = 0$

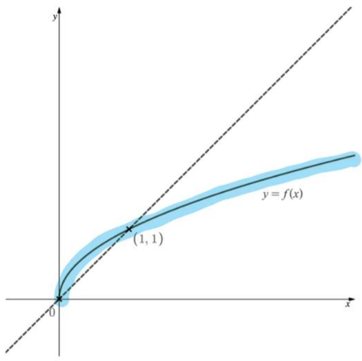
$3x(x+1) = 0$

$\boxed{x = 0 \quad x = -1}$

Q5a

5a

The graph of $y = f(x)$ is shown below.



(a) (i) Use the graph to write down the domain and range of $f(x)$.
(ii) Write down the equation of the dotted line on the graph. [3]

(b) On the diagram above sketch the graph of $y = f^{-1}(x)$. [2]

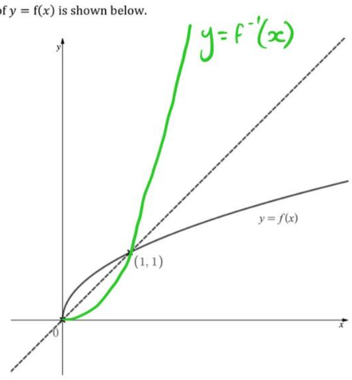
a)
i) DOMAIN $x \geq 0$
RANGE $f(x) \geq 0$

ii) $\boxed{y = x}$

Q5b

5b

The graph of $y = \mathrm{f}(x)$ is shown below.



(a) (i) Use the graph to write down the domain and range of $\mathrm{f}(x)$.

(ii) Write down the equation of the dotted line on the graph.

[3]

(b) On the diagram above sketch the graph of $y = \mathrm{f}^{-1}(x)$.

INVERSE

[2]

b) REFLECT IN $y = x$

Q6a

6a

The functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are defined as follows

$\mathrm{f}(x) = e^{x-2}$ $\quad x \in \mathbb{R}$

$\mathrm{g}(x) = 2 + \ln x$ $\quad x \in \mathbb{R}, x > 0$

(a) Find (i) $\mathrm{fg}(x)$

(ii) $\mathrm{gf}(x)$

COMPOSITE FUNCTIONS
SUB ONE INTO THE OTHER

[3]

(b) Write down $\mathrm{f}^{-1}(x)$ and state its domain and range.

[2]

(c) The graphs of $\mathrm{f}(x)$ and $\mathrm{f}^{-1}(x)$ are drawn on the same axes.
Describe the transformation that would map one graph onto the other.

[2]

a)

i) $fg(x)$

$e^{(\cancel{2} + \ln x) - \cancel{2}}$

$e^{\ln x} = x$

$\boxed{fg(x) = x}$

ii) $gf(x)$

$2 + \ln(e^{x-2})$

$\cancel{2} + x - \cancel{2} = x$

$\boxed{gf(x) = x}$

Q6b

6b

The functions $f(x)$ and $g(x)$ are defined as follows

$f(x) = e^{x-2}$ $\quad x \in \mathbb{R}$
$g(x) = 2 + \ln x$ $\quad x \in \mathbb{R}, x > 0$

(a) Find (i) $fg(x)$
(ii) $gf(x)$

[3]

(b) Write down $f^{-1}(x)$ and state its domain and range.

[2]

(c) The graphs of $f(x)$ and $f^{-1}(x)$ are drawn on the same axes.
Describe the transformation that would map one graph onto the other.

[2]

b) $gf(x) = fg(x) = x$

$g(x)$ AND $f(x)$ ARE INVERSE

$$\boxed{\begin{array}{l} f^{-1}(x) = 2 + \ln x \\ \text{DOMAIN} \quad x \in \mathbb{R} \ \ x > 0 \\ \text{RANGE} \quad f^{-1}(x) \in \mathbb{R} \end{array}}$$

Q6c

6c

The functions $f(x)$ and $g(x)$ are defined as follows

$f(x) = e^{x-2}$ $\quad x \in \mathbb{R}$
$g(x) = 2 + \ln x$ $\quad x \in \mathbb{R}, x > 0$

(a) Find (i) $fg(x)$
(ii) $gf(x)$

[3]

(b) Write down $f^{-1}(x)$ and state its domain and range.

[2]

(c) The graphs of $f(x)$ and $f^{-1}(x)$ are drawn on the same axes.
Describe the transformation that would map one graph onto the other.

[2]

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c) $\boxed{\text{REFLECTION IN LINE } y = x}$